Paper Reference(s)

6664/01

Edexcel GCE

Core Mathematics C2

Advanced Subsidiary

Monday 14 January 2013 – Morning

Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation or integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C2), the paper reference (6664), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 9 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. Find the first 3 terms, in ascending powers of x, in the binomial expansion of

$$(2 - 5x)^6.$$

Give each term in its simplest form.

(4)

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2. $f(x) = ax^3 + bx^2 - 4x - 3$, where a and b are constants.

Given that $(x - 1)$ is a factor of $f(x)$,

(a) show that $a + b = 7$.

(2)

Given also that, when $f(x)$ is divided by $(x + 2)$, the remainder is 9,

(b) find the value of a and the value of b, showing each step in your working.

(4)

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3. A company predicts a yearly profit of £120 000 in the year 2013. The company predicts that the yearly profit will rise each year by 5%. The predicted yearly profit forms a geometric sequence with common ratio 1.05.

(a) Show that the predicted profit in the year 2016 is £138 915.

(1)

(b) Find the first year in which the yearly predicted profit exceeds £200 000.

(5)

(c) Find the total predicted profit for the years 2013 to 2023 inclusive, giving your answer to the nearest pound.

(3)

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4. Solve, for $0 \leq x < 180°$,

$$\cos(3x - 10°) = -0.4,$$

giving your answers to 1 decimal place. You should show each step in your working.

(7)

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5. The circle C has equation

$$x^2 + y^2 - 20x - 24y + 195 = 0.$$

The centre of C is at the point M.

(a) Find

(i) the coordinates of the point M,

(ii) the radius of the circle C.

(5)

N is the point with coordinates (25, 32).

(b) Find the length of the line MN.

(2)

The tangent to C at a point P on the circle passes through point N.

(c) Find the length of the line NP.

(2)

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6. Given that $2 \log_2 (x + 15) - \log_2 x = 6$,

(a) show that $x^2 - 34x + 225 = 0$.

(5)

(b) Hence, or otherwise, solve the equation $2 \log_2 (x + 15) - \log_2 x = 6$.

(2)

7.

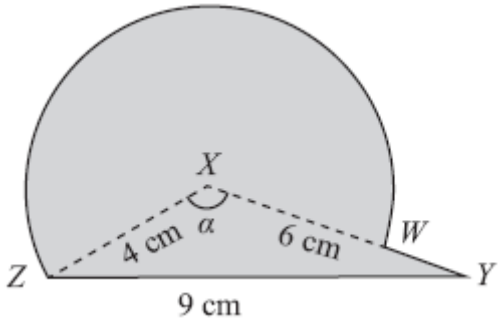

Figure 2

The triangle XYZ in Figure 1 has $XY = 6$ cm, $YZ = 9$ cm, $ZX = 4$ cm and angle $ZXY = \alpha$.
The point W lies on the line XY.

The circular arc ZW, in Figure 1 is a major arc of the circle with centre X and radius 4 cm.

(a) Show that, to 3 significant figures, $\alpha = 2.22$ radians.

(2)

(b) Find the area, in cm^2, of the major sector $XZWX$.

(3)

The region enclosed by the major arc ZW of the circle and the lines WY and YZ is shown shaded in Figure 1.

Calculate

(c) the area of this shaded region,

(3)

(d) the perimeter $ZWYZ$ of this shaded region.

(4)

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8. The curve C has equation $y = 6 - 3x - \frac{4}{x^3}$, $x \neq 0$.

(a) Use calculus to show that the curve has a turning point P when $x = \sqrt{2}$.

(4)

(b) Find the x-coordinate of the other turning point Q on the curve.

(1)

(c) Find $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$.

(1)

(d) Hence or otherwise, state with justification, the nature of each of these turning points P and Q.

(3)

9.

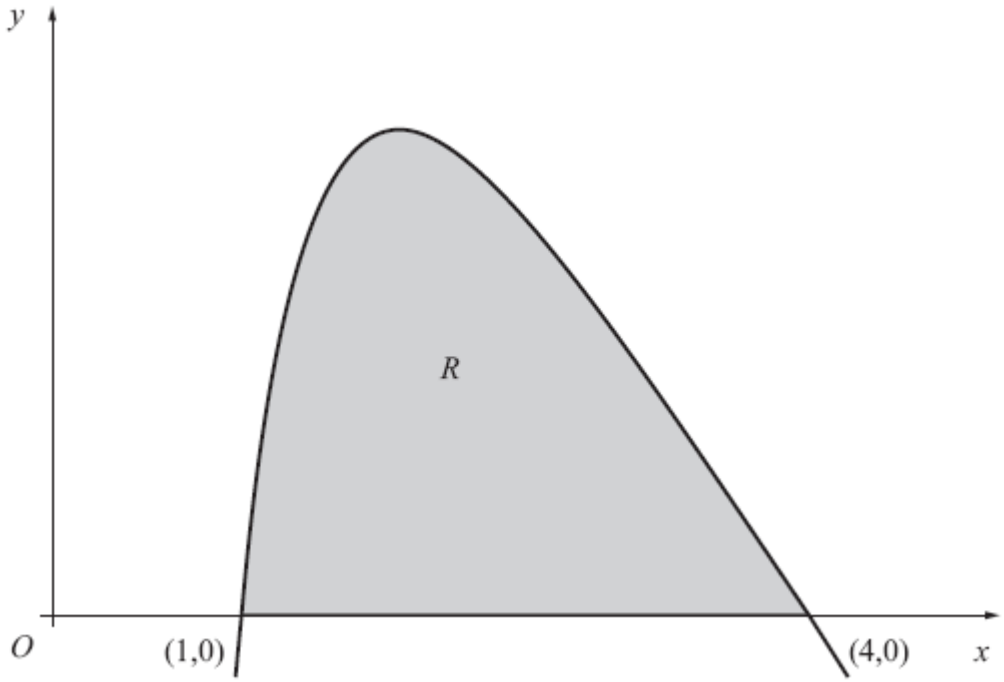

Figure 2

The finite region R, as shown in Figure 2, is bounded by the x-axis and the curve with equation

$$y = 27 - 2x - 9\sqrt{x} - \frac{16}{x^2}, \qquad x > 0.$$

The curve crosses the x-axis at the points (1, 0) and (4, 0).

(a) Copy and complete the table below, by giving your values of y to 3 decimal places.

x	1	1.5	2	2.5	3	3.5	4
y	0	5.866		5.210		1.856	0

(2)

(b) Use the trapezium rule with all the values in the completed table to find an approximate value for the area of R, giving your answer to 2 decimal places.

(4)

(c) Use integration to find the exact value for the area of R.

(6)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme		Marks
1.	$(2-5x)^6$		
	$(2^6 =)\ 64$	Award this when first seen (not $64x^0$)	B1
	$+6\times(2)^5(-5x)+\frac{6\times5}{2}(2)^4(-5x)^2$	Attempt binomial expansion with correct structure for at least one of these terms. E.g. a term of the form: $\binom{6}{p}\times(2)^{6-p}(-5x)^p$ with $p = 1$ or $p = 2$ consistently. Condone sign errors. Condone missing brackets if later work implies correct structure and allow alternative forms for binomial coefficients e.g. ${}^6C_1$ or $\binom{6}{1}$ or even $\left(\frac{6}{1}\right)$	M1
	$-960x$	N**ot** $+-960x$	A1 (first)
	$(+)6000x^2$		A1 (Second) **(4)**
2. (a)	$f(1)=a+b-4-3=0$ **or** $a+b-7=0$	Attempt f(±1)	M1
	$a+b=7$ *	Must be f(1) and = **0** needs to be seen	A1 **(2)**
(b)	$f(-2)=a(-2)^3+b(-2)^2-4(-2)-3=9$	Attempt f(±2) **and uses** f(±2) = 9	M1
	$-8a+4b+8-3=9$ $(-8a+4b=4)$	Correct equation with exponents of (–2) removed	A1
	Solves the **given equation from part (a)** and their equation in a and b from part (b) as far as a =... or b = ...		M1
	$a = 2$ **and** $b = 5$	**Both** correct	A1 **(4)** **[6]**

Question Number	Scheme		Marks
3. (a)	$120000\times(1.05)^3 = 138915*$	Or $120000\times1.05\times1.05\times1.05 = 138915$ Or 120000, 126000, 132300, 138915 Or a = 120000 and $a\times(1.05)^3 = 138915$	B1 **(1)**
(b)	$120000\times(1.05)^{n-1} > 200000$	Allow n or $n-1$ and ">", "<", or "=" etc.	M1
	$\log 1.05^{n-1} > \log\left(\frac{5}{3}\right)$	Takes logs correctly Allow n or $n-1$ and ">", "<", or "=" etc.	M1
	$(n-1>)\ \frac{\log\left(\frac{5}{3}\right)}{\log 1.05}$ or equivalent	Allow n or $n-1$ and ">", "<", or "=" etc. Allow 1.6 or awrt 1.67 for 5/3.	A1
	e.g $(n>)\ \frac{\log\left(\frac{7}{4}\right)}{\log 1.05}$		
	2024	M1: Identifies a calendar year using their value of n or $n-1$	M1 A1 **(5)**
(c)	$\frac{a(1-r^n)}{1-r} = \frac{120000\left(1-1.05^{11}\right)}{1-1.05}$	M1: Correct sum formula with n = 10, 11 or 12 A1: Correct numerical expression with n = 11	M1 A1
	1704814	Cao (Allow 1704814.00)	A1 **(3)** **[9]**

4.	$\cos^{-1}(-0.4) = 113.58\ (\alpha)$	Awrt 114	B1
	$3x-10 = \alpha \Rightarrow x = \frac{\alpha+10}{3}$	Uses their α to find x. Allow $x = \frac{\alpha\pm10}{3}$ **not** $\frac{\alpha}{3}\pm10$	M1
	$x = 41.2$		A1
	$(3x-10=)\ 360-\alpha\ \ (246.4....)$	$360-\alpha$ (can be implied by 246.4...)	M1
	x = 85.5		A1
	$(3x-10=)\ 360+\alpha\ (=473.57....)$	$360+\alpha$ (Can be implied by 473.57...)	M1
	x = 161.2		A1

Question Number	Scheme		Marks
5. (a) (i)	The centre is at (10, 12)	B1: $x = 10$ B1: $y = 12$	B1 B1
(ii)	Uses $(x-10)^2+(y-12)^2 = -195+100+144 \Rightarrow r = ...$ Completes the square for both x and y in an attempt to find r. $(x \pm "10")^2 \pm a$ and $(y \pm "12")^2 \pm b$ and $+195 = 0, (a, b \neq 0)$ Allow errors in obtaining their r^2 but must find square root		M1
	$r = \sqrt{10^2+12^2-195}$	A correct numerical expression for r including the square root and can implied by a correct value for r	A1
	$r = 7$	Not $r = \pm 7$ unless -7 is rejected	A1 **(5)**
(b)	$MN = \sqrt{(25-"10")^2+(32-"12")^2}$	Correct use of Pythagoras	M1
	$MN\left(=\sqrt{625}\right) = 25$		A1 **(2)**
(c)	$NP = \sqrt{("25"^2-"7"^2)}$	$NP = \sqrt{(MN^2-r^2)}$	M1
	$NP\left(=\sqrt{576}\right) = 24$		A1 **(2)** **[9]**
6. (a)	$2\log(x+15) = \log(x+15)^2$		B1
	$\log(x+15)^2 - \log x = \log\dfrac{(x+15)^2}{x}$	Correct use of $\log a - \log b = \log\dfrac{a}{b}$	M1
	$2^6 = 64$ or $\log_2 64 = 6$	64 used in the correct context	B1
	$\log_2\dfrac{(x+15)^2}{x} = 6 \Rightarrow \dfrac{(x+15)^2}{x} = 64$	Removes logs correctly	M1
	$\Rightarrow x^2+30x+225 = 64x$ *or* $x+30+225x^{-1} = 64$ $\therefore x^2-34x+225 = 0$ *	Must see expansion of $(x+15)^2$ to score the final mark.	A1 **(5)**
(b)	$(x-25)(x-9) = 0 \Rightarrow x = 25$ or $x = 9$	M1: Correct attempt to solve the **given** quadratic as far as x =... A1: Both 25 and 9	M1 A1 **(2)** **[7]**

Question Number	Scheme		Marks
7. (a)	$9^2 = 4^2 + 6^2 - 2\times4\times6\cos\alpha \Rightarrow \cos\alpha = ...$	Correct use of cosine rule leading to a value for cos α	M1
	$\cos\alpha = \frac{4^2+6^2-9^2}{2\times4\times6}\left(=-\frac{29}{48}=-0.604..\right)$		
	$\alpha = 2.22$ *	Cso (2.22 must be seen here)	A1
	(NB $\alpha = 2.219516005$)		**(2)**
			(2)
(b)	$2\pi - 2.22 (= 4.06366.....)$	$2\pi - 2.22$ or awrt 4.06	B1
	$\frac{1}{2}\times4^2\times"4.06"$	Correct method for major sector area.	M1
	32.5	Awrt 32.5	A1
			(3)
(c)	Area of triangle = $\frac{1}{2}\times4\times6\times\sin 2.22\left(=9.56\right)$	Correct expression for the area of triangle *XYZ*	B1
	So area required = "9.56" + "32.5"	Their triangle *XYZ* + (part (b) answer or correct attempt at major sector)	M1
	= 42.1 cm^2 or 42.0 cm^2	Awrt 42.1 or 42.0 (Or <u>just</u> 42)	A1
			(3)
(d)	Arc length $= 4\times4.06\left(=16.24\right)$ Or $8\pi - 4\times2.22$	M1: $4\times their\left(2\pi - 2.22\right)$ Or circumference – minor arc A1: Correct ft expression	M1A1ft
	Perimeter = *ZY* + *WY* + Arc Length	9 + 2 + Any Arc	M1
	Perimeter = 27.2 or 27.3	Awrt 27.2 or awrt 27.3	A1
			(4)
			[12]

Question Number	Scheme		Marks
8. (a)	$y = 6 - 3x - \frac{4}{x^3}$		
	$\frac{dy}{dx} = -3 + \frac{12}{x^4} or -3 + 12x^{-4}$	M1: $x^n \rightarrow x^{n-1}$ $(x^{1} \rightarrow x^{0}$ or $x^{-3} \rightarrow x^{-4}$ or $6 \rightarrow 0)$ A1: Correct derivative	M1 A1
	$\frac{dy}{dx} = 0 \Rightarrow -3 + \frac{12}{x^4} = 0 \Rightarrow x = ...$ **or** $\frac{dy}{dx} = -3 + \frac{12}{\sqrt{2}^4}$	$y' = 0$ and attempt to solve for x May be implied by $\frac{dy}{dx} = -3 + \frac{12}{x^4} = 0 \Rightarrow \frac{12}{x^4} = 3 \Rightarrow x = ...$ **or** Substitutes $x = \sqrt{2}$ into their y'	M1
	So $x^4 = 4$ and $x = \sqrt{2}$ **or** $\frac{dy}{dx} = -3 + \frac{12}{\left(\sqrt{2}\right)^4}$ or $-3 + 12\left(\sqrt{2}\right)^{-4} = 0$	Correct completion to answer with no errors by solving their $y' = 0$ or substituting $x = \sqrt{2}$ into their y'	A1
			(4)
(b)	$x = -\sqrt{2}$	Awrt –1.41	B1
			(1)
(c)	$\frac{d^2y}{dx^2} = \frac{-48}{x^5}$ or $-48x^{-5}$	Follow through their first derivative from part (a)	B1ft
			(1)
(d)	An appreciation that either $y'' > 0 \Rightarrow$ a minimum or $y'' < 0 \Rightarrow$ a maximum		B1
	Maximum at P as $y'' < 0$		B1 cso
	Minimum at Q as $y'' > 0$		B1 cso
			(3)
			[9]

Question Number	Scheme	Marks
9. (a)	$y = 27 - 2x - 9\sqrt{x} - \frac{16}{x^2}$	
	6.272 , 3.634	B1, B1 **(2)**
(b)	$\frac{1}{2}\times\frac{1}{2}$ or $\frac{1}{4}$	B1
	$....\left\{(0+0)+2\left(5.866+"6.272"+5.210+"3.634"+1.856\right)\right\}$ | Need {} or implied later for A1ft	M1A1ft
	$\frac{1}{2}\times 0.5\left\{(0+0)+2\left(5.866+"6.272"+5.210+"3.634"+1.856\right)\right\}$	
	= $\frac{1}{4}\times 45.676$	
	= 11.42	A1 cao **(4)**
(c)	$\int y\,dx = 27x - x^2 - 6x^{\frac{3}{2}} + 16x^{-1}(+c)$ | M1: $x^n \to x^{n+1}$ on any term A1: $27x - x^2$ A1: $-6x^{\frac{3}{2}}$ A1: $+16x^{-1}$	M1A1A1A1
	$\left(27(4)-(4)^2-6(4)^{\frac{3}{2}}+16(4)^{-1}\right)$ $-\left(27(1)-(1)^2-6(1)^{\frac{3}{2}}+16(1)^{-1}\right)$ | Attempt to subtract either way round using the limits 4 and 1. Dependent on the previous M1	dM1
	= (48 – 36)	
	12	A1 cao **(6)**
		[12]